REINFORCED CONCRETE ROOF
FOR THE
ROCKWELL ST. CAR HOUSE. C. C. R. CO.

C. S. MILLARD

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concrete roof for the

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DESIGN OF A REINFORCED CONCRETE ROOF FOR THE ARCHER AVENUE AND ROCKWELL STREET CAR HOUSE OF THE CHICAGO CITY RAILWAY COMPANY.

A THESIS PRESENTED BY

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to the

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OF THE

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The Archer avenue and Rockwell street Car House of the Chicago City Railway Company is located at the intersection of the streets which give it its name. By reference to the plan of the building shown on Plate I, it will be seen that the car house is a structure 490 ft. in length by 310 ft. in width. To reduce the risk of fire, the building is divided into eight (8) bays, separated by brick fire walls. One (1) bay known as the service bay, is devoted to the offices, boiler room, shop and wreck wagon room. The remainder of the building is used entirely for car storage.

In designing the roof of the car house, it was first decided to use steel trusses and a reinforced concrete slab roof. This method of construction was used at the 38th street & Cottage Grove avenue Car House. In order to make this construction fireproof, however, it was necessary to fireproof the trusses with metal lath and a prepared plaster. This construction was found to be more expensive and less satisfactory than reinforced concrete girders.

THEORY OF THE REINFORCED CONCRETE TEE BEAM

As shown on Plate VIII, the girders are both tee shaped and rectangular in section. On account of an 8 ft. continuous skylight in the center of each bay, the girders are rectangular in section between skylight curbs. In designing the girders, the formulae given in Turneaure & Maurer's "Principles of Reinforced Concrete Construction" were used. These formulae are based upon the tension in steel and are approximate. As stated by the authors, on page 80, there are no satisfactory formulae based on

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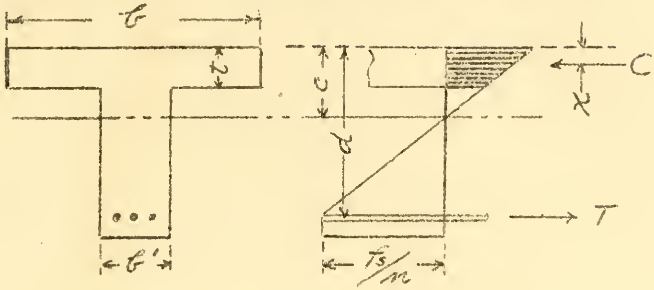
the compression in the concrete.

The theory of the tee beam is as follows:



Notation

b = width of flange
d = effective depth of beam
b' = width of web
t = thickness of flange
c = depth of neutral axis below top of flange
x = depth of resultant compression below top of flange
f_s = unit tension in steel
f_c = unit compression in concrete
e_s = unit elongation of steel due to f_s
e_c = unit shortening of concrete due to f_c
E_s = modulus of elasticity of steel
E_c = modulus of elasticity of concrete
n = ratio of $E_s \div E_c$
T = total tension in steel
C = total compression in the concrete
M_s = resisting moment as determined by steel
M_c = resisting moment as determined by concrete
M = bending moment
k = ratio of the depth of the neutral axis below the top to d
j = ratio of the arm of the resisting couple to d
A = area of cross section of steel
p = steel ratio $A \div b'd$

Since the modulus of elasticity is the ratio of the stress to the deformation we have

$$e_s = \frac{f_s}{E_s} \quad \text{and} \quad e_c = \frac{f_c}{E_c}$$

In order to derive a theory of flexure for concrete

beams, two (2) assumptions are necessary, 1st Navier's hypothesis, that a plane section of a beam before bending is plane after bending; 2nd that the material obeys Hooke's Law that stress is proportional to strain.

The following deductions result from the above assumptions: First, that unit deformations at any section of a beam, are proportional to their distances from the neutral axis. Secondly, that the unit stresses are also proportional from the neutral axis.

From the above

With a comparatively thin slab, the neutral axis is generally below the bottom of the flange. It will be considered that all the compression in the concrete is carried by the flange. Unless the neutral axis is coincident with the bottom of the flange, this assumption is not true. However, the error is on the side of safety.

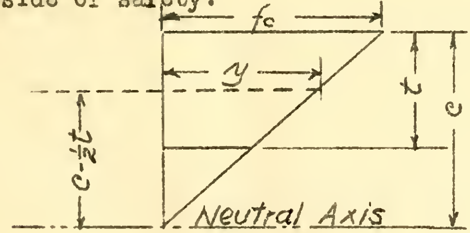


FIG. 2

The average unit compression $y = \frac{f_c}{c}(c - \frac{1}{2}t)$. This result is obtained by proportion in the triangle shown in Figure 2

$$y : f_c = (c - \tfrac{1}{2}t) : c$$

$$cy = f_c(c - \tfrac{1}{2}t) \qquad y = \frac{f_c}{c}(c - \tfrac{1}{2}t)$$

The total compression $= \frac{f_c}{c}(c - \frac{1}{2}t)bt$

As the total compression equal the total tension

$$f_s A = \frac{f_c}{c}\left(c - \frac{1}{2}t\right) bt \qquad (2)$$

By eliminating $\frac{f_s}{f_c}$ between equations (1) and (2) and solving for c

$$c = \frac{2nd\ A + bt^2}{2(nA + bt)} \qquad \text{where } n = \frac{E_s}{E_c} \qquad (3)$$

By taking moments about the top of the flange, the value of x, the distance to the center of gravity of the compressive area is found

$$x = \frac{3c - 2t}{2c - t} \times \frac{t}{3} \qquad (4)$$

With the resisting moment depending upon the steel,

$$M_s = Af_s(d - x) \qquad (5)$$

$(d-x)$ is the aim of the resisting couple.

From this equation $f_s = \frac{M_s}{A(d-x)}$ (6)

With the resisting moment depending upon the concrete

$$M_c = \frac{f_c}{c}\left(c - \frac{1}{2}t\right) bt\,(d - x).$$

From equation (1) substituting n for $\frac{E_s}{E_c}$

$$f_c = \frac{f_s}{n} \times \frac{c}{d-c} \qquad (7)$$

The application of these formulae will be considered under the subject of the design of reinforced concrete girders.

THEORY OF RECTANGULAR REINFORCED CONCRETE BEAMS

This theory is given in Turneaure & Maurer's "Principles of Reinforced Concrete Construction". It is based on the linear

variation of the compression in the concrete. The notation is the same as that given for the tee beam.

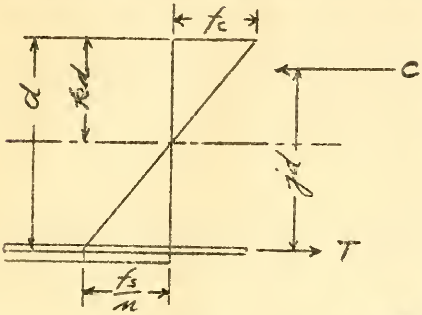


FIG 3

As the unit deformations vary as their distances from the neutral axis,

$$\frac{e_s}{e_c} = \frac{d-kd}{kd}$$

$$e_s = \frac{f_s}{E_s}$$

and $e_c = \frac{f_c}{E_c}$ (See the theory of the tee beam)

$$\frac{\frac{f_s}{E_s}}{\frac{f_c}{E_c}} = \frac{e_s}{e_c} = \frac{d-kd}{kd} = \frac{1-k}{k}$$

Substituting the value of $n = \frac{E_s}{E_c}$.

$$\frac{f_s}{nf_c} = \frac{1-k}{k} \qquad (8)$$

The total tension on the section being equal to the total compression

$$f_s A = \frac{1}{2} f_c \quad bkd \qquad (9)$$

Eliminating $\frac{f_s}{f_c}$ and substituting p for its value $\frac{A}{bd}$, the following equations result,

$$n\frac{(1-k)}{k} = \frac{k}{2p} \qquad 2pn(1-k) = k^2$$

$$k = \sqrt{2pn + (pn)^2} - pn \qquad (10)$$

This formula shows that the depth of the neutral axis depends entirely upon the percentage of steel and the ratio of the moduli of elasticities. The center of gravity of the compressive area, is at a distance of $\frac{1}{3}kd$ from the top of the section. The lever arm of the resisting couple is equal to jd.

$$jd = d - \frac{1}{3}kd \qquad j = 1 - \frac{1}{3}k \qquad (11)$$

If the resisting moment depends upon the steel,

$$M_s = f_s A \times jd = f_s pjbd^2 \qquad (12)$$

If the beam is over reinforced the compression in the concrete is the determining factor, and the resisting moment has the following value,

$$M_c = \frac{1}{2}f_c bkd \times jd = \frac{1}{2}f_c kjbd^2 \qquad (13)$$

Solving equations (12) and (13) for bd^2,

$$bd^2 = \frac{M}{f_s p j} \qquad (14)$$

$$bd^2 = \frac{M}{\frac{1}{2}f_c k j} \qquad (15)$$

THEORY OF SHEAR AND BOND STRESSES

This theorectical discussion is taken from the same authors as the preceding theories.

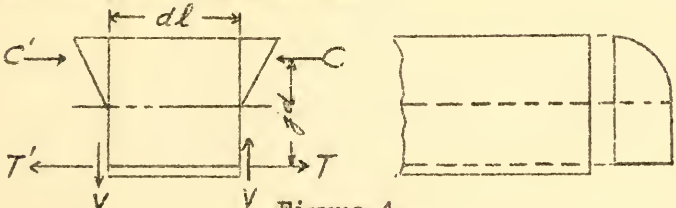


Figure 4.

$C = C' =$ Total compression
$T = T' =$ Total tension
$V =$ Total vertical shear
$v =$ Unit horizontal shear
$dl =$ Short length of beam
$jd =$ Arm of resisting couple
$b' =$ Width of beam

In this discussion the tension area in the concrete is neglected. The difference,between the tensile stresses T and T',will be equal to the total shear on any horizontal plane between the steel and the neutral axis. The unit shear would therefore be equal to $\frac{T'-T}{b'(dl)}$ or

$$v = \frac{T'-T}{b'(dl)} \qquad (16)$$

Equating the moments of the shears,

$$V\,(dl) = (T'-T)\,j\,d$$

$$T'-T = \frac{V(dl)}{j\,d} \qquad (17)$$

Substituting the value $(T'-T)$ in equation (16)

$$v = \frac{V}{b'jd} \qquad (18)$$

Taking the average value as .875 and substituting in (16)

$$v = \frac{8}{7}\,\frac{V}{b'd} \qquad (19)$$

Formula (16) is applicable to tee beams as well as rectangular beams. If $(d - \frac{1}{2}t)$ is substituted for jd, the error will be very small, and on the side of safety.

$$v = \frac{V}{b'(d - \frac{1}{2}t)} \qquad (20)$$

As previously stated $(T' - T)$ is the shear on any horizontal plane. Per unit of length, this shear will equal $\frac{T' - T}{dl}$

If U equals the bond stress per lineal inch, then

$$U = \frac{T' - T}{d} \quad \text{and}$$

$$U = \frac{V}{j\,d} \qquad (21)$$

By dividing U by the sum of the perimeters of the steel sections, the bond stress per unit area of steel is obtained.

SPACING OF GIRDERS

A series of calculations each similar to those which follow, lead to the adoption of 16 ft. as the most economical spacing of the girders.

BENDING MOMENT

In order to determine the effect of settlement of the supports of a continuous slab roof an investigation was made of a roof slab of five (5) spans. The slab was assumed to have a thickness of 4-1/2". The weight of the roof was 55# per sq.ft. The theory used in determining the bending moments of the continuous spans, was the graphic method developed by Prof. C. E. Greene in his "Trusses and Arches, Graphic Method Part II". The four equations necessary for determining the pier ordinates for a beam of five (5) spans are as follows:

$$H\left[(2\underline{la}+\underline{lb})y_1 + \underline{lb}y_2\right] = 6H\left(\frac{Aa}{\underline{Ia}}+\frac{Bb'}{\underline{Ib}}\right)+6EI\left(\frac{va}{\underline{Ia}}+\frac{vc}{\underline{Ib}}\right) \qquad (22)$$

$$H\left[\underline{lc}y_1 + 2(\underline{lb} + \underline{lc})y_2 + \underline{lc}y_3\right] = 6H\left(\frac{Bb}{\underline{Ib}} + \frac{Cc'}{\underline{Ic}}\right)-6EI\left(\frac{vb}{\underline{Ib}} + \frac{vd}{\underline{Ic}}\right) \qquad (23)$$

$$H\left[\underline{l}cy_2 + 2(\underline{l}c + \underline{l}d)\,y_3 + \underline{l}dy_4\right] = 6H\left(\frac{Cc}{\underline{lc}} + \frac{Dd'}{\underline{ld}}\right) - 6EI\left(\frac{vc}{\underline{lc}} + \frac{ve}{\underline{ld}}\right) \quad (24)$$

$$H\left[\underline{l}dy_3 + 2(\underline{l}d + \underline{l}e)y_4\right] = 6H\left(\frac{Dd}{\underline{ld}} + \frac{Ee'}{\underline{le}}\right) - 6EI\left(\frac{vd}{\underline{ld}} + \frac{vf}{\underline{le}}\right) \quad (25)$$

A reference to Plate II, and the following notation will explain the purpose of these equations:

H = Pole distance in the stress diagrams, = 500#

$\underline{la}$ $\underline{lb}$ $\underline{lc}$ $\underline{ld}$ and $\underline{le}$ represent the lengths of the five (5) spans, in this case all equal to 16 ft.

y_1 y_2 y_3 and y_4 represent the ordinates under the supports in the equilibrium polygons. The purpose of the equations is the determination of the values of these ordinates. They are the only unknown quantities and when their values are found, the closing lines of the polygons may be drawn.

A, B, C, D, E are the areas inclosed by the equilibrium polygons and the horizontal line R_1 R_6.

These areas are all equal to 40.5 sq.ft. in this problem.

a and a' represent the distances from the center of gravity of the area A to the left and right-hand supports respectively.

bb' cc', etc., represent similar distances for areas B, C, etc.

Where the supports are not on the same level, the deflections of the points of supports $\underline{va}$, $\underline{vb}$, $\underline{vc}$, etc., below or above some line of reference must be taken into consideration. In order to determine the effect on the adjacent spans, if one (1) support should settle, R_3 was considered to have settled distances 1", 1/2" and 1/4"; all the other supports being on the same level. In considering equation (22) involving $\underline{la}$ and $\underline{lb}$, the

tangent or reference line passes through R_2. Therefore $\underline{va} = 0$ and $\underline{vc}$ has a value of + 1", with 1" of settlement. For equation (23) the tangent passes through R_3. The valuesof $\underline{vb}$ and $\underline{vd}$ =-1". E is the modulus of elasticity of the concrete, which was taken as 2,000,000. I is moment of inertia of the section which was equal to 91.

Determination of y_1 y_2 y_3 & y_4 with the supports on the same level.

Substituting in equation (22)

$$64y_1 + 16y_2 = 6\left(\frac{324}{16} + \frac{324}{16}\right)$$

$$4y_1 + y_2 = 15.15$$

$$y_1 = \frac{15.15 - y_2}{4}$$

Equation (23)

$$y_1 + 4y_2 + y_3 = 6\left(\frac{324}{16} + \frac{324}{16}\right)$$

$$y_1 + 4y_2 + y_3 = 15.15$$

Equation (24)

$$y_2 + 4y_3 + y_4 = 15.15$$

Equation (25)

$$y_3 + 4y_4 = 15.15$$

Solving these equations, the following values were obtained:

$$y_1 = y_4 = 3.19 \text{ ft.}$$

$$y_2 = y_3 = 2.39 \text{ ft.}$$

Having the valuesof y_1 y_2 y_3 and y_4 it was only necessary to lay these distances off at the points of support as shown on

Plate II. The closing lines were drawn, connecting the extremities of the y ordinates.

1" settlement of R_3

With the tangent passing through R_2, $\underline{va} = 0$ $\underline{vc} = +1"$

Substituting in equation (22)

$$500 \times 16(4y_1 + y_2) = 16 \times 15.15 \times 500 + \frac{6 \times 2{,}000{,}000 \times 91}{144} \times \frac{1}{16 \times 12}$$

$$4y_1 + y_2 = 15.15 + \frac{12{,}000{,}000 \times 91}{144 \times 16 \times 500} \times \frac{1}{16 \times 12}$$

$$4y_1 + y_2 = 20.08$$

Equation (23) tangent through R_3

$$\underline{vb} = \underline{vd} = -1"$$

$$y_1 + 4y_2 + y_3 = 5.29$$

Equation (24) tangent through R_4

$$\underline{vc} = +1" \quad \underline{ve} = 0$$

$$y_2 + 4y_3 + y_4 = 20.08$$

Equation (25) tangent through R_5

$$\underline{vd} = \underline{vf} = 0$$

$$y_3 + 4y_4 = 15.15$$

Solving the 4 equations for the 4 unknowns:

$y_1 = 5.31$ ft.

$y_2 = -1.19$ ft.

$y_3 = 4.65$ ft.

$y_4 = 2.62$ ft.

Having these values the closing lines were drawn, as in the previous onse. For a settlement of 1/2" for R_3, the values

of the y ordinates are as follows:

y_1 = 4.25 ft.
y_2 = .6 ft.
y_3 = 3.52 ft.
y_4 = 2.91 ft.

By comparing the values of y_1 it is seen that a 1/2" settlement of R_3 increases y_1 from 3.19 ft. to 4.25 ft. or there is an increment of 1.06 ft. An additional settlement of 1/2", increases y_1 to 5.31 ft. on an increment of 1.06. This relationship is true for y_2, y_3 and y_4. Having established the values of y for any given settlement, it is possible to draw the closing lines for any other settlement without further calculations. For each span, the closing lineshave a common point of intersection as will be seen by referring to Plate 11.

With the closing lines drawn, to obtain the bending moments, it is only necessary to scale the ordinates from the closing lines to the equilibrium polygons, multiplying these ordinates by H. On Plate III is a table showing the maximum positive and negative bending moments, and also the points of inflection for various conditions of the supports. From the table it is seen that for the intermediate panels the maximum positive moment is $\frac{wl^2}{21.7}$ and the maximum negative moment $\frac{wl^2}{12.9}$

In designing the roof slab, on account of the possible lack of full continuity, a compromised value for the positive bending moment was used. This value was midway between $\frac{wl^2}{8}$ for a simple span and $\frac{wl^2}{21.7}$ for a continuous span, or $\frac{wl^2}{14.4}$

For the negative moment a value of $\frac{wl^2}{10}$ was used. This value would take care of a possible settlement of 1/4".

DESIGN OF THE REINFORCED CONCRETE ROOF SLAB

The thickness of the slab was taken as 4-1/2", giving a dead load of 55# per sq. ft. With a snow load of 25# per sq. ft., the total load becomes 80# per sq. ft.

Reinforcement to resist positive bending moment. Width of slab 12".

$$M = \frac{wl^2}{14.4} = \frac{80x256x12}{14.4} = 17,100 \text{ in. } \#$$

Formula (12) gives the required area of steel

$$A = \frac{M}{f_s j d}$$

j depends upon k, p and n. The value of j can be calculated from the formulae (10) and (11) previously given. It is much simpler to use the diagram given on Plate V which was taken from Turneaure & Maurer's "Principles of Reinforced Concrete Construction" page 57.

Assume $p = .0083$ $\quad n = \frac{Es}{Ec} = \frac{30,000,000}{2,000,000} = 15$

From the diagram for $p = .0083$ and $n = 15$, $j = .875$ and $k = .390$ d the effective depth is 3.5" $f = 16,000\#$ per sq. in.

$$A = \frac{17,100}{16000x.875x3.5} = .35 \text{ sq. in.}$$

Considering only the concrete above the center of the reinforcement, the area per ft. of width equals 12x3.5 = 42 sq. in.

$$p = \frac{.35}{42} = .00833$$

The value assumed of p is correct. This percentage can only be obtained by trial. The spacing for 1/2" square rods is as follows: The area of a 1/2" square rod is .25 sq. in. The number of rods required per ft. $= \frac{.35}{.25} = 1.4$ $\qquad \frac{12"}{1.4} = 8\text{-}1/2"$ spacing.

8" spacing was actually used in the roof.

Reinforcement to resist negative bending moment.

Assume p = .012 j therefore equals .85 and k equals .445

$$M = \frac{wl^2}{10} = \frac{80x256x12}{10} = 24{,}600 \text{ in. \#}$$

$$A = \frac{24600}{16000x.85x3.5} = .516 \text{ sq. in.}$$

$$p = \frac{.516}{42} = .0123$$

Spacing of 1/2" square rods

$$\frac{.516}{.25} = 2.06 \text{ rods per ft.} \quad \frac{12"}{2.06} = 5.83"$$

6" spacing was used on the roof.

BOND

The stress on the bottom rods resisting the positive bending moment, becomes zero at the points of inflection. Taking the shear at the point which averages 4 ft. from the supports the following bond stresses result:

$$U = \frac{V}{jd}$$

$$V = 80 \times 4 = 320\# \qquad j = .875 \qquad d = 3.5$$

$$U = \frac{320}{.875 \times 3.5} = 104.5\#$$

For 8" spacing the sum of the perimeters per ft. width of slab = 2x1.5 = 3 inches.

$$\frac{104.5}{3} = 34.8\# \text{ per sq. inch}$$

For the top or negative reinforcing rods

$$U = \frac{640}{.85 \times 3.5} = 215\#$$

For 6" spacing the sum of the perimeters = 2x2" = 4 inches

$$\frac{215\#}{4} = 53.75\# \text{ per sq. inch}$$

These values for bond stresses are not high. For Johnson corrugated bars, which were used in the roof, the experiments of the University of Illinois given in their Bulletin #8, show that the average ultimate bond stress to be 595# per square inch for 1/2" square bars.

SHEARING STRESSES

From formula (19)

$$v = \frac{8V}{7\ b\ d}$$

The maximum shear V = 640#

$$v = \frac{8 \times 640}{7 \times 12 \times 3.5} = 17.4\# \text{ sq. inches}$$

END PANEL

The calculations for the end panels are exactly similar to those for the intermediate spans, except for the value of the bending moment. The value of M was taken as $\frac{Wl^2}{8}$. The length

of the end panels is 13 ft. and the spacing of the bars 5-1/2".

TEMPERATURE AND SHRINKAGE STRESSES

The amount of reinforcement necessary to resist the temperature and shrinkage stresses can only be roughly estimated. With a temperature range of 50° F. the stress in the steel would be 50x.0000065xE = 9750# per sq. inch, where the coefficient of expansion for steel is .0000065 per degree F. Large cracks can not form in the concrete, unless the steel is stretched beyond its elastic limit. Assume the concrete to have a tensile strength of 200# per sq. inch, and the elastic limit of the high carbon steel is 60000# per sq. inch

$$p = \frac{200}{60000-9750} = .004$$

To resist temperature and shrinkage stresses transverse to the direction of the main reinforcement, 1/2" square corrugated bars were arbitrarily spaced 1 ft. apart. This corresponds to a value of p of .006. In the direction of the span, additional reinforcement was added by running every third one of the top rods the entire length of span, and every second one of the bottom rods, as shown on Plate VIII.

SKYLIGHT CURB

Loads:

Assume the curb to carry 2 ft. of the adjacent slab:

Skylight	125#	per	ft.
Snow	50#	"	"
Slab	160#	"	"
	335#	per	ft.

$$M = \frac{wl^2}{8} = \frac{335x256x12}{8} = 129000 \text{ inch lbs.}$$

With a width of curb of 8"; and with p = .0125

$$d^2 = \frac{129000}{8x.85x16000x.0125} = 95$$

where k = .45 and j = .86, d = 9.75"

Total depth 11.75"

Reinforcement of two (2) - 7/8" round rods.

DESIGN OF THE REINFORCED CONCRETE GIRDERS

With a girder spacing of 16 ft., the roof load to be carried amounts to 16x80# or 1280# per ft. or girder. The dead load of the girder must first be assumed. Having made several trial calculations for the required depth with a given width, the dead load can be determined exactly, and the depth refigured. Assume the width of the beam to be one (1) ft. Owing to the pitch of the roof, which is 1" per ft. the dead load varies from a minimum at the support to a maximum in the center.

As each bay of the car house has an eight (8) ft. continuous skylight, the tee shaped girder is only effective from the points of support up to the skylight. On Plate IV the dead and live loads of the girder are shown.

For the Tee section, a certain width of slab may be considered as a part of the girder. Just how much of the slab is effective, it is impossible to determine. Various formulae and rules have been proposed for determining b the breadth of slab. The Prussian Regulations of 1904, give the permissible width as less than one-third (1/3) the length of the span; the French and British rules, less than three-quarter (3/4) spacing

of the beams. If these rules are applied to the case under consideration, it would mean that anything under 12 ft. could be considered as the effective width of the slab. In the June, 1908 Bulletin of the Corrugated Bar Company, a table of formulae for tee beams is given. For 4-1/2" slab, with an effective girder depth of 24", the formula is $b = .152\, b' l - \frac{1}{2} b'$. b' = breadth of beam = 12." l = length of span = 37 ft. b = 62.5". The value of b used in designing the girders was 4 ft.

$$\text{From equation (3)} \quad C = \frac{2nd\ A + bt^2}{2(nA + bt)}$$

$$\text{and } X = \frac{3c - 2t}{2c - t} \times \frac{t}{3} \qquad (4)$$

In order to facilitate the solution of these formulae, a diagram was made (Plate VI), which gives values of C and X for various values of A and d. The quantities n, b and t are constant for this particular problem. n = 15 b = 48" and 4.5". The intersection of the lines representing the depth of the girder and the area of the steel, is horizontally projected to the right giving the proper value of C. This value of C when projected horizontally to an intersection with the curve on the right-hand side of the diagram, will give the proper value of X vertically above the point of intersection.

The bending moments for designing the various sections of the girder are found from the diagram on Plate IV. Sections were taken two (2) ft. apart corresponding with the points at which the loads were assumed to be applied. Section #1 Ordinate = 82" H = 40000. Bending moment = 82 x 40000 = 3,280,000 inch

pounds.

$$bd^2 = \frac{M}{f_s \; p \; j} \qquad \text{Formula (12)}$$

$$bd^2 = \frac{M}{\frac{1}{2} f_c \; k \; j} \qquad \text{Formula (13)}$$

Applying (12)

$$d^2 = \frac{3,280,000}{16000x.0115x.85x12} = 1748$$

$$d = 41.8''$$

Applying (13)

$$d^2 = \frac{3,280,000}{350x.44x.85x12} = 2083$$

$$d = 45.5''$$

From formula (12) the depth is dependent upon the steel and in (13) the compression in the conrete is the controlling factor. The maximum value obtained was 45.5 inches for the effective depth. In the design the effective depth was taken as 45" and the total depth 48". In solving the above formulae, the following values were used $f_s = 16000\#$, $f_c = 700\#$, $p = .0115$, $j = .85$, $k = .44$. The value of p was found after several trials. It corresponds to a reinforcement of 6.12 sq. inches or eight (8) - 7/8" square rods. Having assumed p, the corresponding values of j and k were found from Plate V, where n = 15. Having assumed eight (8) 7/8" square rods for the reinforcement, the stresses in the steel and concrete will be calculated.

Section #3. The depth at this point is the result of several trials. Having determined upon the effective depth of 36.5", the depths of all sections were fixed by the slope of 1" to the foot of the roof. Referring to Plate IV , C = 12.5" and X = 2.08" for d = 36.5" and A = 6.2. sq.in.

Substituting these values in (6) and (7)

$$fs = \frac{Ms}{A(d-X)} = \frac{3,160,000}{6.2x34.42} = 14800\#$$

$$fc = \frac{f_s \times c}{n(d-c)} = \frac{14800x12.5}{15x24} = 514\#$$

Section #4 d = 34.5" C = 11.8" X = 2.07" A = 6.2sq.

$$fs = \frac{3,080,000}{6.2x32.43} = 15300\#$$

$$fc = \frac{15300x11.8}{15x22.7} = 530\#$$

Section #5 d = 32.5" C = 11.5" X = 2.065" A = 6.2 sq.in.

$$fs = \frac{2840,000}{6.2x30.435} = 15000\#$$

$$fc = \frac{15000x11.5}{15x21} = 548\#$$

Section #6 d = 30.5" C = 10.7" X = 2.05" A = 6.2 sq.in.

$$fs = \frac{2,520,000}{6.2x28.45} = 14300\#$$

$$fc = \frac{14300x10.7}{15x19.8} = 515\#$$

Section #7 d = 28.5" C = 10.2" X = 2.03" A = 6.2 sq.in.

$$fs = \frac{2,080,000}{6.2x26.47} = 12700\#$$

$$fc = \frac{12700x10.2}{15x18.3} = 472\#$$

The stresses being low at this section with eight (8) 7/8" square bars, two (2) bars were turned up at section #7a, 1 ft. beyond section #7. The stresses for section #7a with six (6) bars are as follows:

d = 27.5" C = 8.4" X = 1.97" A = 4.6 sq. in.

$$fs = \frac{1,750,000}{4.6x25.53} = 14900\#$$

$$fc = \frac{14900x8.4}{15x19.1} = 438\#$$

Section #8 d = 26.5" C = 8.2" X = 1.97" A = 4.6 sq.in.

$$fs = \frac{1,560,000}{4.6x24.53} = 13820\#$$

$$fc = \frac{13820x8.2}{15x18.3} = 413\#$$

Section #9 d = 24.5" C = 6.2" X = 1.83" A = 3.1 sq.in.

$$fs = \frac{1,000,000}{3.1x22.67} = 14200\#$$

$$fc = \frac{14200x6.2}{15x18.3} = 321\#$$

The above stresses indicate that it is safe to turn up the second pair of bars at this point, leaving four (4) bars to run horizontally the entire length of the girder.

Section #10 d = 22.5" C = 6.0" X = 1.81" A = 3.1 sq.in.

$$fs = \frac{240,000}{3.1x20.69} = 3740\#$$

$$fc = \frac{3740x6}{15x16.5} = 90\#$$

SHEARING STRESSES

The shearing stresses are tabulated on Plate VII . The total shear V was taken from the shear diagram on Plate IV. Equation (20) gives the unit shear for a tee beam $v = \frac{V}{b'(d-\frac{1}{2}t)}$. As an example of calculating the area required for the stirrups, section #8 will be considered. The method of determining the number and size of the stirrups is only an approximation and not exact. It is generally considered that a unit shearing stress of 30# per square inch in the concrete is safe. When the stresses are greater than this, it is necessary to provide steel to take care of the shear. In making the calculations for the girder, steel was provided to take care of the unit shearing stresses above 30# per square inch. In section #8, $v = \frac{22600}{291} = 77.60\#$ per sq. in. $77.60\# - 30\# = 47.60\#$

The amount of steel required in a length equal to d or 26.5" =

$$\frac{(v-30)b'(d-\frac{1}{2}t)}{f_s} = \frac{47.60 \times 291}{16000} = .865 \text{ sq. in.}$$

The stirrups used in the girder were made up of two (2) U shaped sections of 1/2" round corrugated bars. Each stirrup had a total cross sectional area of .7854 sq. in. If .865 sq. inches of steel were required in a length of 26.5 in., the theorectical spacing of the stirrups would be 24 inches. In the design, the theorectical spacing was not adhered to, as the stirrups were placed much closer than required by the calculations. The reason for this, was the uncertainty of any determinations of this kind. Furthermore vertical stirrups were used, which are not as effective as inclined rods. Vertical stirrups, however, are easier

to place in position and interfere less with the filling of the forms with concrete. No account was taken of inclined reinforcing bars in resisting shear.

FORMS

Details of the forms are shown on Plate VIII. Long Leaf Yellow Pine was used for the forms except for the 1" boards and some of the small braces. For roof boards, Hemlock was used and Norway Pine for the braces. To facilitate handling the forms, girder bottoms were made in three (3) sections; and the sides in five (5) sections. The 6"x6" posts, two (2) for each girder, carried a load of 16,500# each. The actual cross section of the posts was 30.25 sq. in., giving a unit compression of 545# per sq. in. This load was distributed over a plank foundation having an area of 16 sq. ft. The average settlement of the posts when the forms were filled was 3/8 of an inch. To take care of this settlement the girders were given a camber of 1". When the roof was completed, it was found that all the heavy form lumber of 2" thickness or more was practically as good as new, after having been used three (3) times. The lighter material had no value after being used the same number of times.

CONSTRUCTION

The concrete for the roof was a mixture of 1 : 2-1/2 : 5, the stone being small enough to pass through a 1/2" ring. Two (2) machines were used for mixing the concrete, which was hoisted to the roof in wheel-barrows. The roof slab was made of 4" of concrete on top of which was placed 1/2" of 1 : 3 cement mortar.

The mortar was used in order to make the roof watertight, as the roof was not covered with any waterproof material. Joints between one day's work and the next were placed half-way between girders where the shear was zero. For the concrete that was placed in good weather, the slab forms and girder sides were left in place for ten (10) days, and the girder bottoms and posts for three (3) weeks. For work done in freezing weather the slab forms were left in place for four (4) weeks and the girder posts for six (6) weeks. Photographs taken by the writer, when acting as Engineer in Charge of construction, are shown on Plates IX, X and XI.

EXPANSION JOINTS

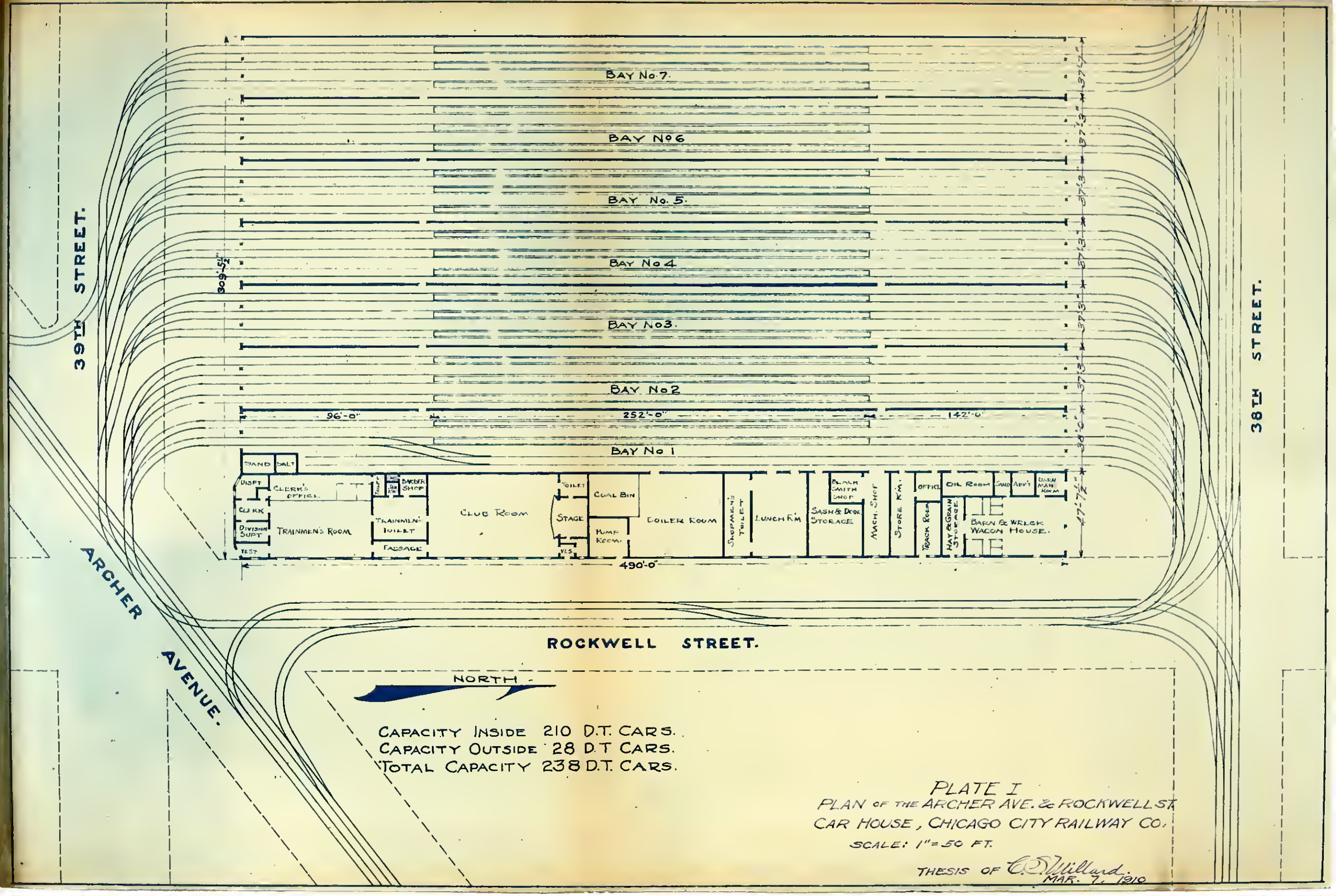
Provisions for expansion were made at all points of bearing. Strips of #18 gauge galvanized iron were placed over all walls to break the bond between the concrete slab and the brick work On Plate VIII, details are shown of the expansion joints at the end walls. With a temperature range of 50 degrees F. the theoretical expansion and contraction would be nearly 2 inches for a roof 490 ft. long.

Expansion per degree F. = .000006

Expansion for 50 deg. F. = .0003

.0003 x 490 = .147 ft. = 1.76"

Sufficient space was allowed over the walls for an expansion of two (2) inches. The actual movement of the slab between summer and winter averaged 3/4 of an inch. As the car house is heated in winter, the temperature of the roof slab does not vary 50° F. during the year.



CAPACITY INSIDE 210 D.T. CARS.
CAPACITY OUTSIDE 28 D.T. CARS.
TOTAL CAPACITY 238 D.T. CARS.

PLATE I
PLAN OF THE ARCHER AVE. & ROCKWELL ST.
CAR HOUSE, CHICAGO CITY RAILWAY CO.
SCALE: 1" = 50 FT.

THESIS OF C. S. Willard.
MAR. 7. 1910

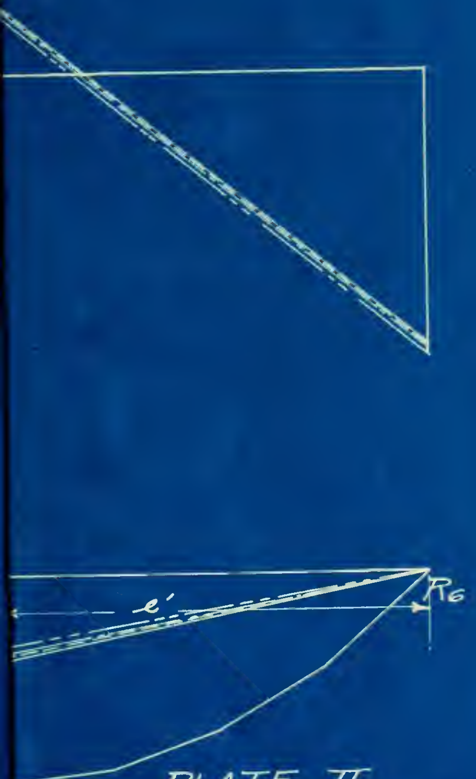


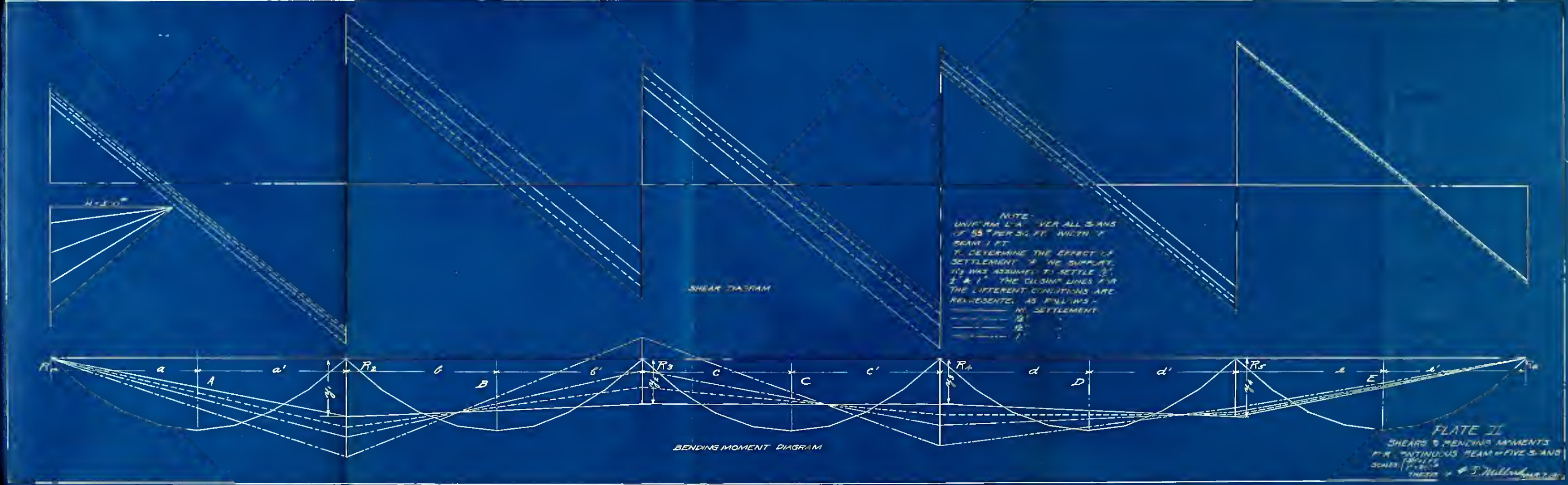
PLATE II

SHEARS & BENDING MOMENTS

R CONTINUOUS BEAM OF FIVE SPANS

LES: { 3/8" = 1 FT.
1" = 200#

THESIS OF C.S. Millard MAR. 7. 1910



NOTE:-
UNIFORM LOAD OVER ALL SPANS OF 55# PER SQ. FT. WIDTH OF BEAM 1 FT.
TO DETERMINE THE EFFECT OF SETTLEMENT OF ONE SUPPORT, R_3 WAS ASSUMED TO SETTLE ¼", ½" & 1". THE CLOSING LINES FOR THE DIFFERENT CONDITIONS ARE REPRESENTED AS FOLLOWS:-

————	NO SETTLEMENT
— — — —	¼" "
—·—·—	½" "
—··—··—	1" "

PLATE II

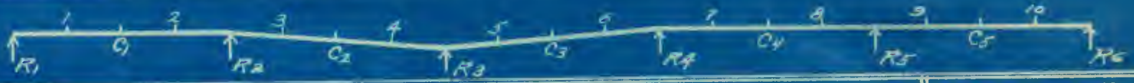
SHEARS & BENDING MOMENTS FOR CONTINUOUS BEAM OF FIVE SPANS

SCALES { [illegible]

THESIS OF E. S. Holbrook MAR 7 19[illegible]

EFFECT OF SETTLEMENT OF ONE SUPPORT ON A CONTINUOUS BEAM OF FIVE SPANS

LENGTH OF ALL SPANS 16'-0". WIDTH OF BEAM 1'-0". UNIFORM LOAD 55# PER SQ. FT. = w
+ = POSITIVE BENDING MOMENT. − = NEGATIVE. C_1 C_2 ETC. = POINT OF MAX. + BENDING MOMENT

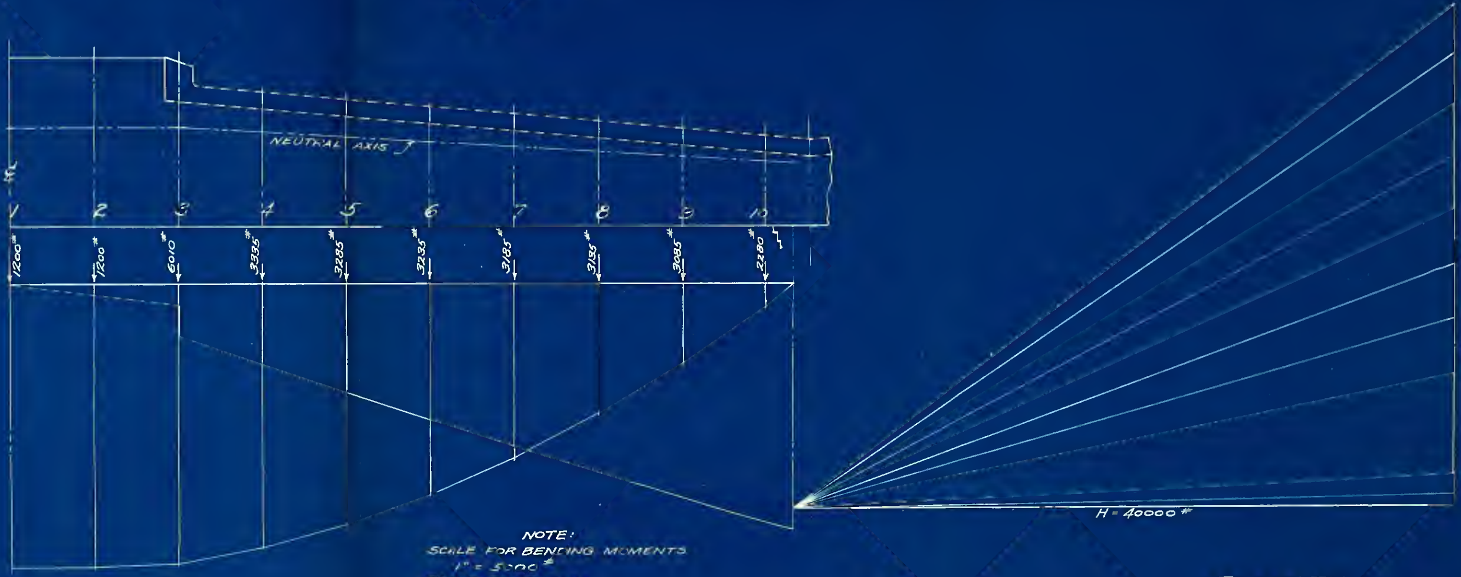


POINT	BENDING MOMENTS COEFFICIENTS OF wL^2				REACTIONS COEFFICIENTS OF wL			
	NOR-MAL	SETTLEMENT OF R_3			NOR-MAL	SETTLEMENT OF R_3		
		1/4"	1/2"	1"		1/4"	1/2"	1"
R_1	0	0	0	0	.396	.380	.365	.334
C_1	$+\frac{1}{12.8}$	$+\frac{1}{14}$	$+\frac{1}{15.4}$	$+\frac{1}{19.4}$				
R_2	$-\frac{1}{9.7}$	$-\frac{1}{8.5}$	$-\frac{1}{7.2}$	$-\frac{1}{5.8}$	1.135	1.192	1.260	1.380
C_2	$+\frac{1}{30.7}$	$+\frac{1}{24.7}$	$+\frac{1}{18.4}$	$+\frac{1}{12.7}$				
R_3	$-\frac{1}{12.9}$	$-\frac{1}{29.3}$	$-\frac{1}{46}$	$+\frac{1}{26.3}$	.969	.880	.798	.604
C_3	$+\frac{1}{21.7}$	$+\frac{1}{19.2}$	$+\frac{1}{16.8}$	$+\frac{1}{11.9}$				
R_4	$-\frac{1}{12.9}$	$-\frac{1}{10.9}$	$-\frac{1}{8.6}$	$-\frac{1}{6.6}$	.969	1.041	1.110	1.250
C_4	$+\frac{1}{30.7}$	$+\frac{1}{41.5}$	$+\frac{1}{52.4}$	$+\frac{1}{122}$				
R_5	$-\frac{1}{9.7}$	$-\frac{1}{10}$	$-\frac{1}{10.5}$	$-\frac{1}{12.5}$	1.135	1.105	1.072	1.020
C_5	$+\frac{1}{12.8}$	$+\frac{1}{12.4}$	$+\frac{1}{12.2}$	$+\frac{1}{11.9}$				
R_6	0	0	0	0	.396	.401	.403	.412

POINT	DISTANCE FROM NEAREST REACTION TO PT. OF INFLECTION				RATIO OF DISTANCE OF PT. OF INFLECTION TO SPAN			
	NOR-MAL	SETTLEMENT OF R_3			NOR-MAL	SETTLEMENT OF R_3		
		1/4"	1/2"	1"		1/4"	1/2"	1"
1								
2	3.38 FT.	3.90 FT.	4.50 FT.	5.60 FT.	.211	.244	.281	.350
3	4.25	4.50	4.75	5.10	.266	.281	.297	.319
4	3.45	2.25	1.10	—	.216	.141	.069	—
5	3.20	2.05	1.00	—	.200	.128	.063	—
6	3.20	3.60	3.95	4.50	.200	.225	.247	.281
7	3.45	4.33	5.25	7.75	.216	.271	.328	.485
8	4.25	4.50	4.65	5.25	.266	.281	.291	.328
9	3.38	3.15	3.05	2.80	.211	.197	.191	.175
10								

PLATE III

THESIS OF E. S. Hillard
MAR. 7. 1910



NOTE:
SCALE FOR BENDING MOMENTS
1" = 5000 #
SCALE FOR SHEARS
1" = 10000 #

PLATE IV

BENDING MOMENTS
FOR
REINFORCED CONCRETE GIRDER
SPAN 39 FEET.
SCALES: { 1" = 5000 # & 10000 #
½" = 1 FT.
THESIS OF C. S. Millard MAR. 7, 1910.

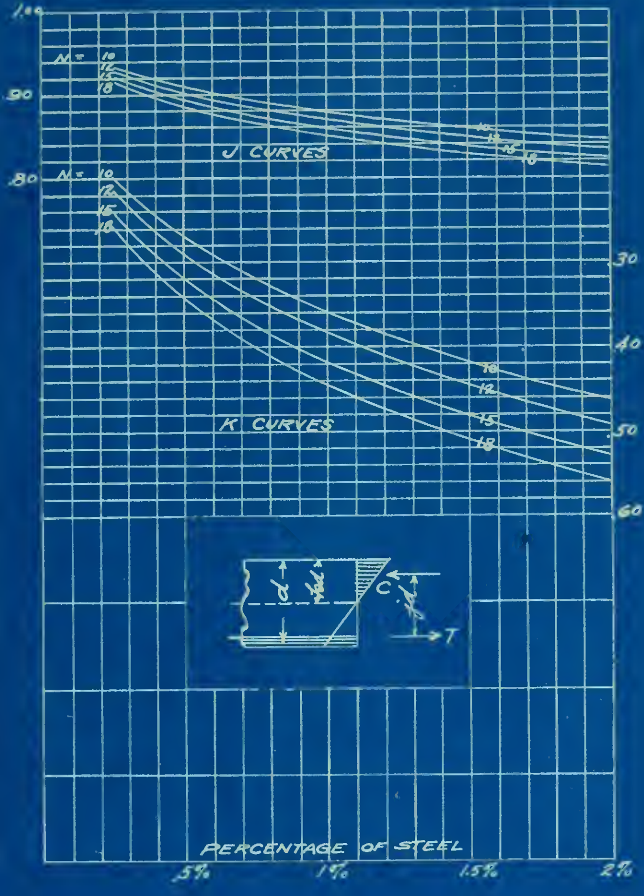
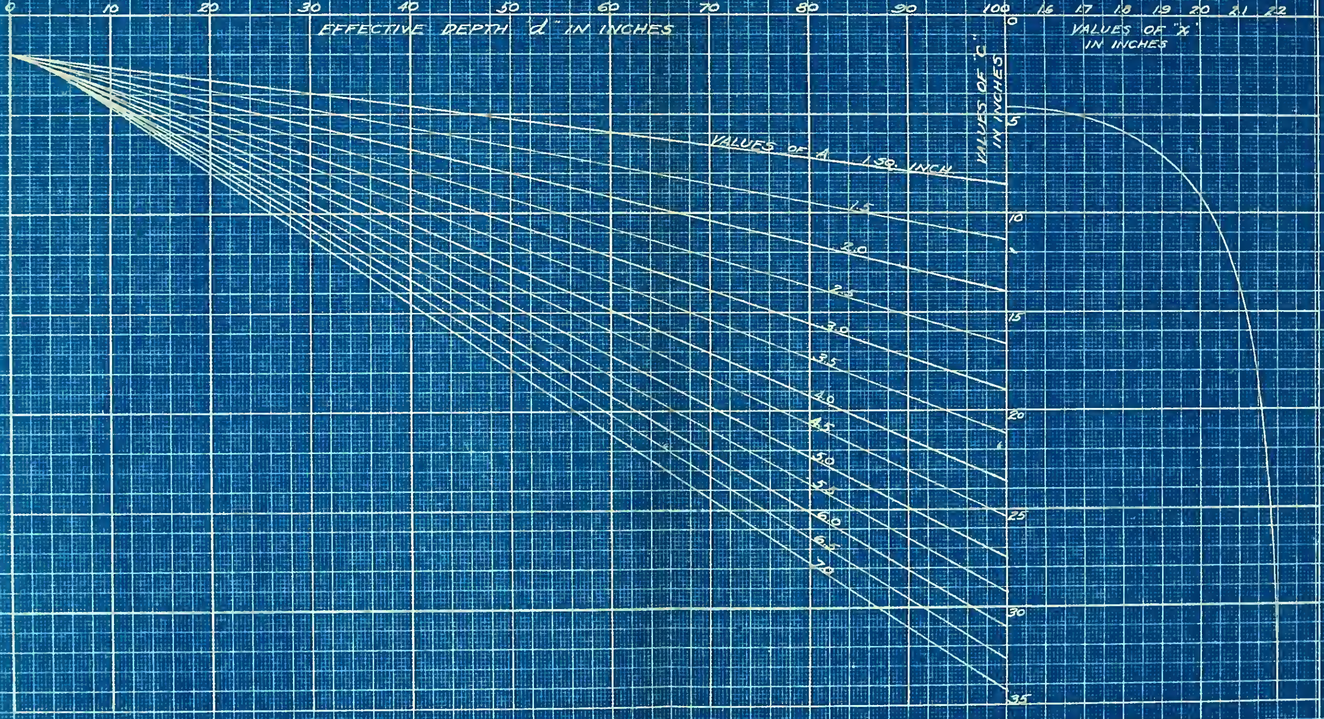


PLATE V
VALUES OF "K" AND "J"
FROM TURNEAURE & MAURER'S
"PRINCIPLES OF REINFORCED CONCRETE
CONSTRUCTION"
THESIS OF C S Millard
MAR 7, 1910.



NOTE: THIS DIAGRAM IS ONLY APPLICABLE TO A TEE BEAM WHERE THE CONSTANTS HAVE THE FOLLOWING VALUES – $n = 15$ $b = 48"$ $t = 4.5"$

PLATE VI

DIAGRAM FOR SOLUTION OF EQUATIONS

$$C = \frac{2ndA + bt^2}{2(nA + bt)} \text{ AND } x = \frac{3c - 2t}{2c - t} \times \frac{t}{3}$$

THESIS OF C. S. Willard MAR. 7. 1910

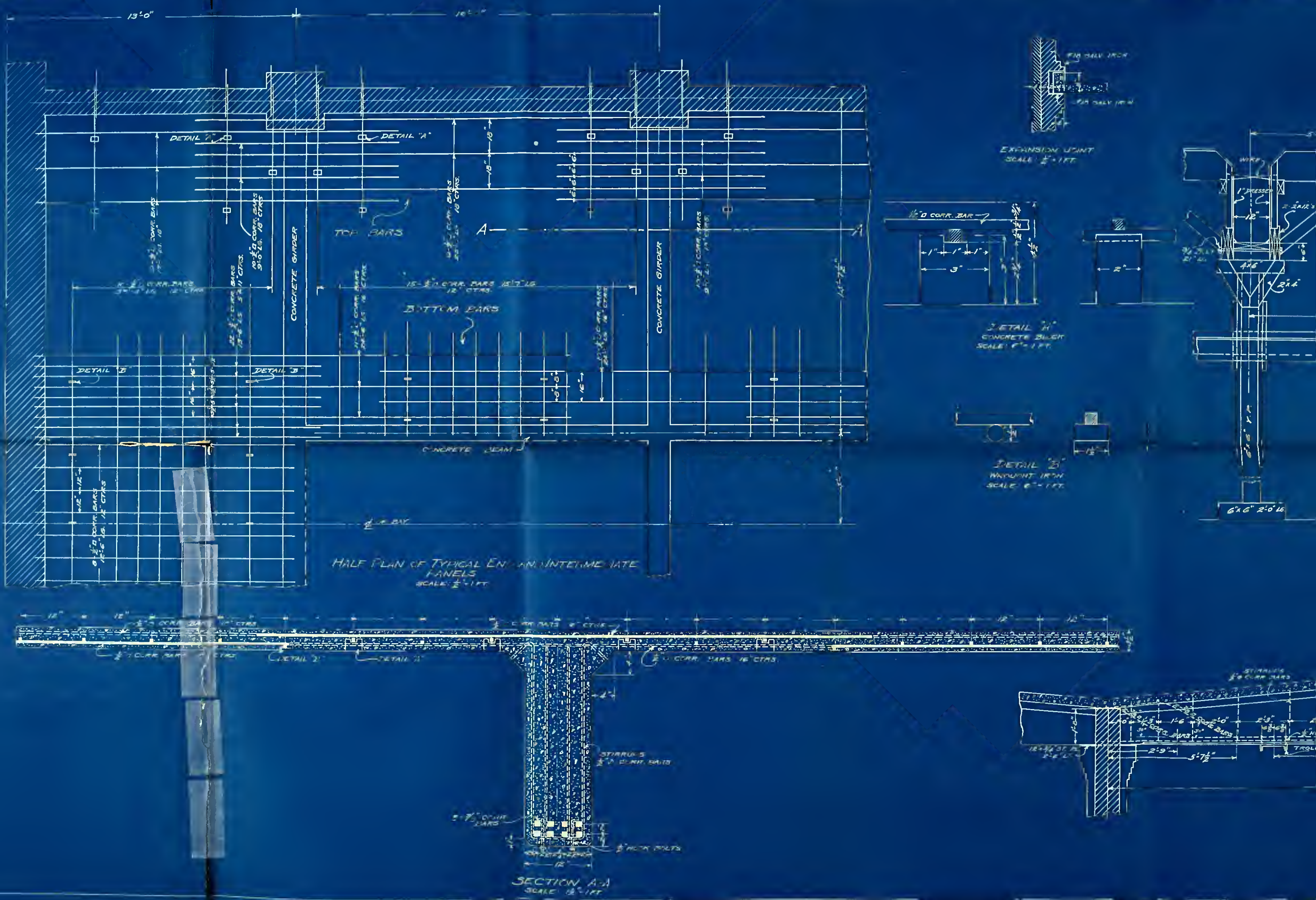
SHEARING STRESSES

SECTION	EFFECTIVE DEPTH d	TOTAL SHEAR V	$b'(d-\frac{1}{2}t)$	UNIT SHEAR $v=\frac{V}{b'(d-\frac{1}{2}t)}$	AREA OF STEEL "A", REQUIRED FOR SHEAR IN LENGTH d	THEORETICAL SPACING OF STIRRUPS.
1	45.0"	0		0		
2	45.0"	1200#		2.5 # SQ.IN.		
3	36.5"	6200#	411	15.10 # "		
4	34.5"	9500#	387	24.60 "		
5	32.5"	13000#	363	35.80 "	.1315 SQ.IN.	194"
6	30.5"	16200#	339	47.80 "	.378 "	63"
7	28.5"	19400#	315	61.60 "	.623 "	36"
8	26.5"	22600#	291	77.60 "	.865 "	24"
9	24.5"	25400#	267	95.10 "	1.085 "	17"
AT SUPPORT	21.5"	29350#	231	127.00 "	1.400 "	12"

NOTE: SECTIONS 1 AND 2 ARE RECTANGULAR, SHEAR CALCULATED FROM FORMULA $v=\frac{8V}{7bd}$

PLATE VII

THESIS OF C. S. Mullard
MAR. 7, 1910.



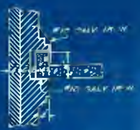
HALF PLAN OF TYPICAL END AND INTERMEDIATE PANELS
SCALE: ¼"=1 FT.

SECTION A-A
SCALE: ½"=1 FT.

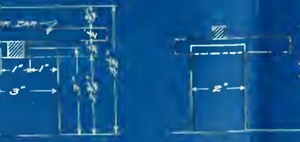
EXPANSION JOINT
SCALE: ½"=1 FT.

DETAIL "H"
CONCRETE BLOCK
SCALE: 6"=1 FT.

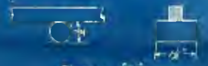
DETAIL "B"
WROUGHT IRON
SCALE: 6"=1 FT.



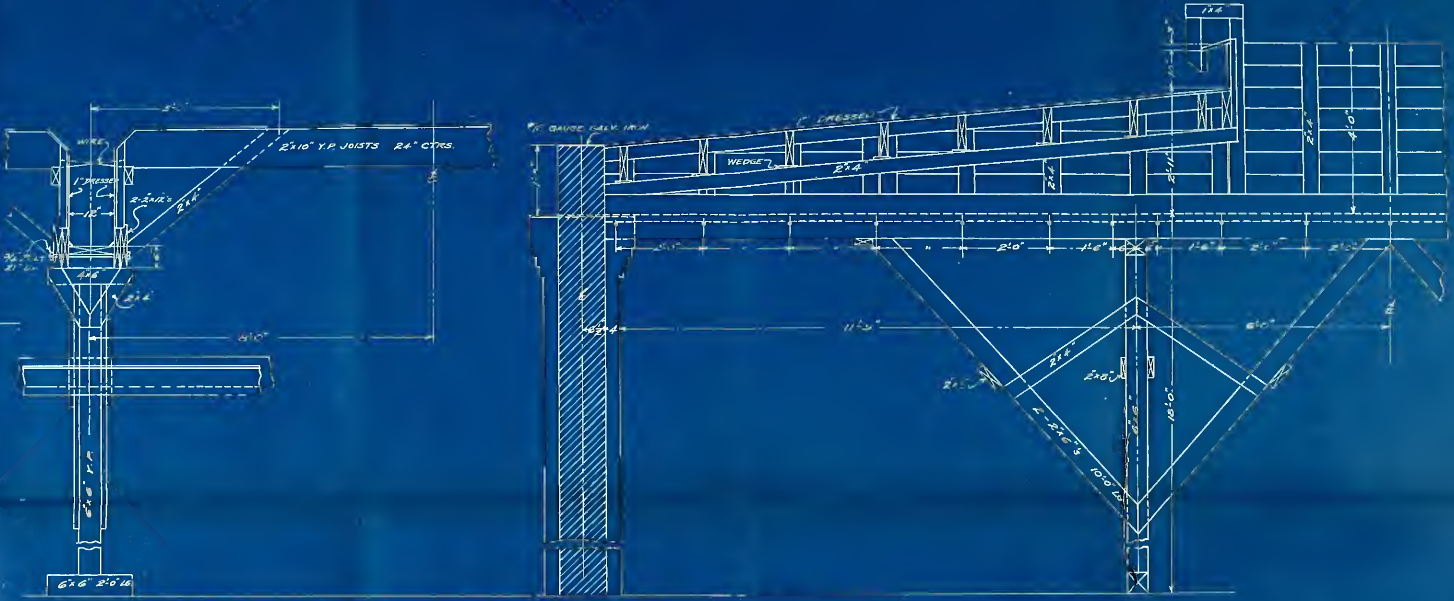
EXPANSION JOINT
SCALE: 1½" = 1 FT.



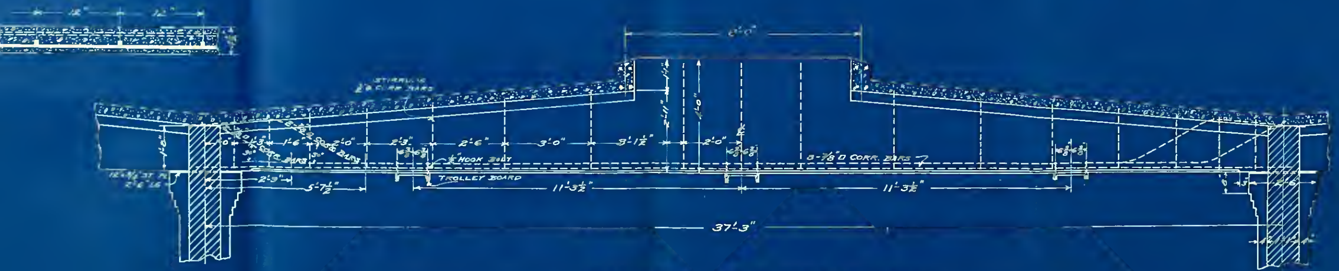
DETAIL "A"
CONCRETE BLOCK
SCALE: 6" = 1 FT.



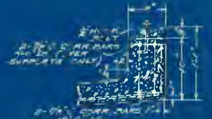
DETAIL "B"
WROUGHT IRON
SCALE: 6" = 1 FT.



ROOF SLAB & GIRDER FORMS
SCALE: ¾" = 1 FT.



ELEVATION OF REINFORCED CONCRETE
GIRDER
SCALE: ½" = 1 FT.

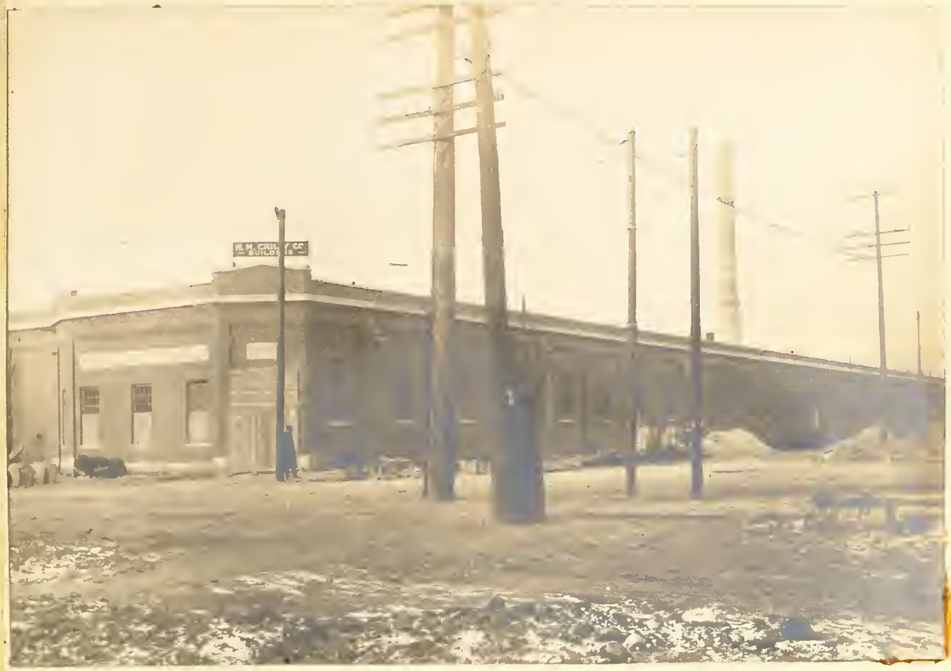


SECTION OF SKYLIGHT
CURB
SCALE: 1" = 1 FT.

PLATE VIII

REINFORCED CONCRETE DETAILS,
ARCHER AVE. & ROCKWELL ST. CAR
HOUSE, CHICAGO CITY RAILWAY CO.

THESIS OF
MAR. 5,

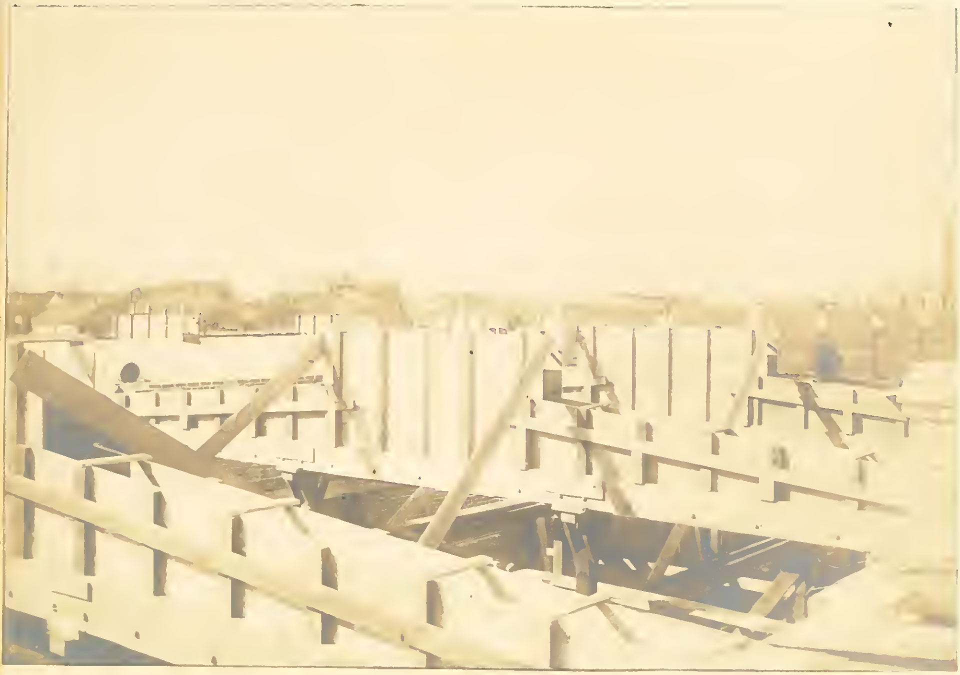


S.E. CORNER OF THE ARCHER & ROCKWELL CAR HOUSE.

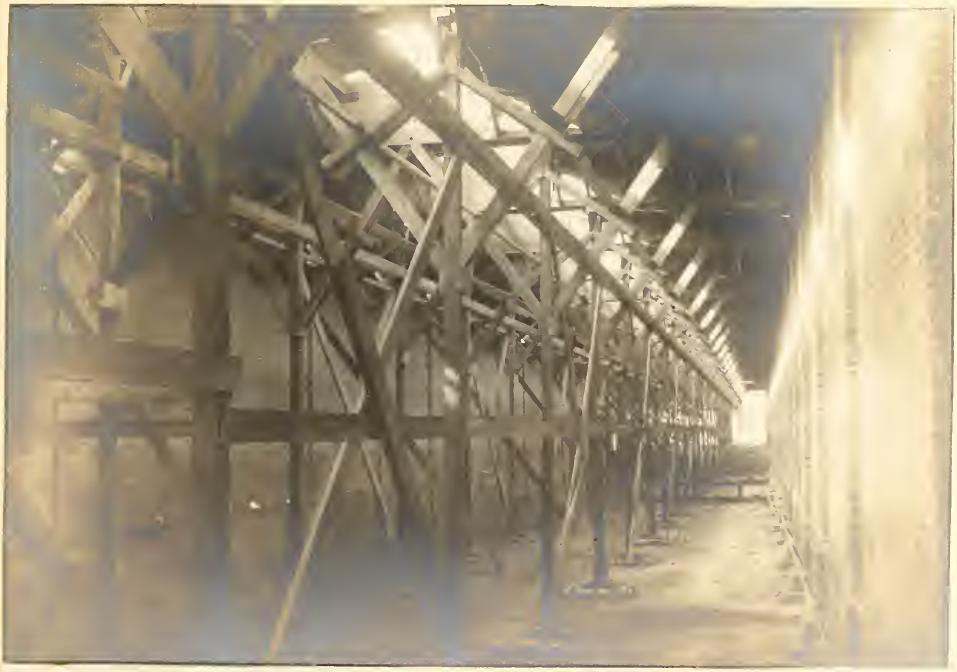
INTERIOR OF A BAY

PLATE IX

2



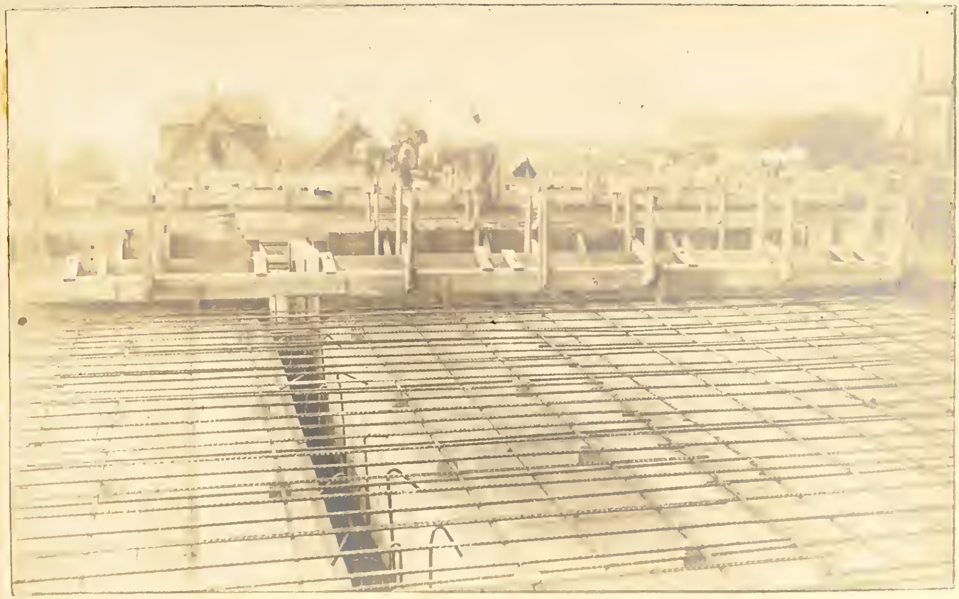
GIRDER FORMS.



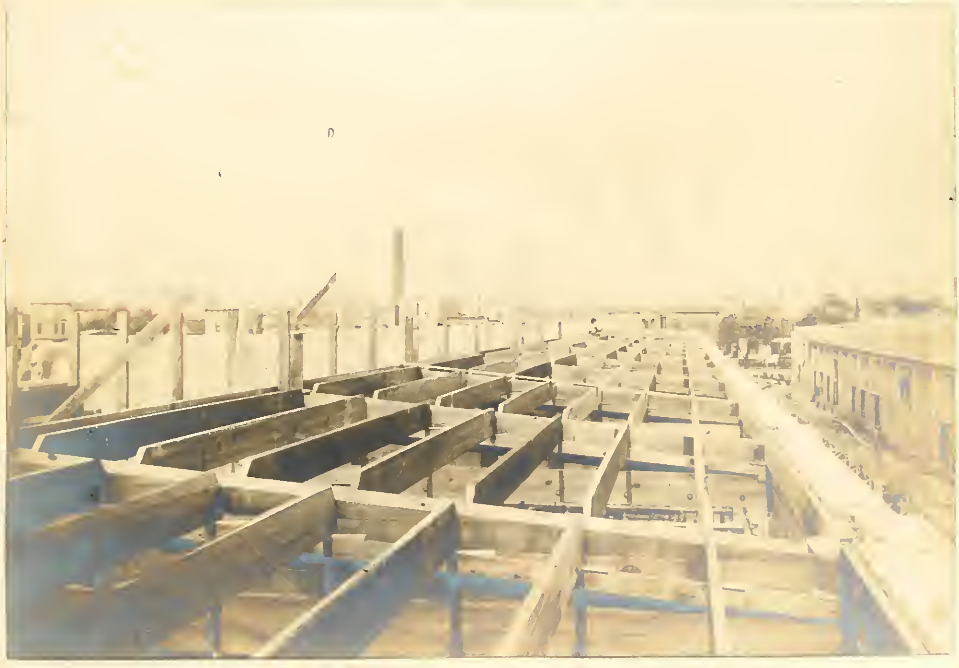
ROOF FORMS FROM BELOW

PLATE X

3



GIRDER & ROOF SLAB REINFORCEMENT



JOISTS FOR ROOF SLAB FORMS

PLATE XI

22153
T
q695.6 M611

Millard, C. S.

Reinforced concrete roof for
Rockwell st.car house C. C.
co.

DEMCO 216

T
q695.6 22153 M611

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